Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-10-18

Conceptual questions

Question 1. As you know, the (filled-in) sphere of radius R centered at the origin has the equation $x^2 + y^2 + z^2 \leq R^2$.

(a) How would you describe this region in cylindrical coordinates?

(b) How would you describe this region in spherical coordinates?

Question 2. Consider the region enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$. Set up integrals which compute the volume of this region in all 6 integration orders (the various permutations of $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$).

Question 3. Let R and H be positive real numbers. Sketch the solid (in xyz-space) whose volume is given by the integral below.

$$\int_0^R \int_0^{2\pi} \int_{Hr/R}^{H} r\,\mathrm{d}z\,\mathrm{d}\theta\,\mathrm{d}r.$$

(Note that R is not one of the integration variables, so it is totally fine for it to be in the outer bounds.)

Computations

Problem 1. Verify that integration gives you the correct formula (in terms of R, H) for the solid in Question 3 above.

Problem 2. Derive the formula $V = \frac{4}{3}\pi R^3$ for the volume of a sphere of radius R. Try using both spherical and cylindrical coordinates.

Problem 3. Derive the formula $SA = 4\pi R^2$ for the surface area of a sphere of radius R, using §15.5 methods. Note that this is equal to $\mathrm{d}V/\mathrm{d}R$. Does this make sense geometrically?